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NOTE ON CAUCHY'S INTEGRAL TEST.

By M. B. PORTER, University of Texas.

It does not seem to have been generally noticed that series which consist of a monotonic decreasing sequence of terms of the same sign lend themselves to calculation as readily as those whose signs alternate provided the integral $\int_n^\infty u_n dn$ is sufficiently simple. We have at once, if the terms are all positive,

$$\int_n^\infty u_n dn \leq u_n + u_{n+1} + u_{n+2} + \dots \text{etc.} \leq u_n + \int_n^\infty u_n dn.$$

Thus if to the sum of the first $n-1$ terms we add $\frac{u_n}{2} + \int_n^\infty u_n dn$, our approximation will not differ from the sum of the series by more than $u_n/2$, that is, by less than half of the first term we omit.

Thus to calculate the series $\sum_1^\infty n^{-2}$ the correction after $n-1$ terms is $\frac{1}{2n^2} + \frac{1}{n}$, and the calculated value is in error not more than $\frac{1}{2n^2}$. This fact can be used to calculate with a minimum of labor such slowly convergent series as those represented by the logarithmic scales of DeMorgan.

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

346. Proposed by E. B. ESCOTT, Professor of Mathematics, University of Michigan.

Solve completely by quadratics:

$$\frac{2x}{1-x^2}=y; \quad \frac{2y}{1-y^2}=z; \quad \frac{2z}{1-z^2}=w; \quad \frac{2w}{1-w^2}=x.$$

Solution by the PROPOSER.

Let $x=\tan\phi$, then $y=\tan 2\phi$, $z=\tan 4\phi$, $w=\tan 8\phi$, $x=\tan 16\phi=\tan\phi$,
 $\tan 16\phi - \tan\phi = 0$, or $\frac{\sin 15\phi}{\cos 16\phi \cdot \cos\phi} = 0$.

If $\cos\phi = \infty$, $\tan\phi = \pm i$.

If $\sin 15\phi = 0$, $\phi = \frac{n\pi}{15}$ ($n=0, 1, 2, \dots, 14$).

$\therefore x = \pm i, 0, \tan \frac{\pi}{15}, \tan \frac{2\pi}{15}, \dots, \tan \frac{14\pi}{15},$

$$= \pm i, 0, \pm \frac{\sqrt{[10+2\sqrt{5}]} - \sqrt{(15)} + \sqrt{(3)}}{\sqrt{[30+6\sqrt{5}]} + \sqrt{(5)} - 1}, \pm \frac{\sqrt{(15)} + \sqrt{(3)} - \sqrt{[10-2\sqrt{5}]}}{\sqrt{[30-6\sqrt{5}]} + \sqrt{(5)} + 1},$$

$$\pm \frac{\sqrt{[10-2\sqrt{5}]}}{\sqrt{(5)} + 1}, \pm \frac{\sqrt{[10+2\sqrt{5}]} + \sqrt{(15)} - \sqrt{(3)}}{\sqrt{[30+6\sqrt{5}]} - \sqrt{(5)} + 1}, \pm \sqrt{(3)},$$

$$\pm \frac{\sqrt{[10+2\sqrt{5}]}}{\sqrt{(5)} - 1}, \pm \frac{\sqrt{(15)} + \sqrt{(3)} + \sqrt{[10-2\sqrt{5}]}}{\sqrt{[30-6\sqrt{5}]} - \sqrt{(5)} - 1}.$$

(These values are taken from Hobson's *Plane Trigonometry*.)

Professor J. Scheffer gave a partial solution, following the method in the above solution. Mr. Spuner sent in a similar solution, with results given in decimals instead of radicals.

347. Proposed by GUSTAVE JACOBSON, A. M., Public Accountant, Chicago, Ill.

A corporation needing some additional capital for a short term of years, issues \$300,000 of debenture bonds carrying 6% interest, and payable 1/5 each year for 5 years. Coupons are attached to the bonds maturing every six months; the bonds are sold at 90 flat. What average rate of interest does the company pay for the money, including interest on interest?

Solution by THEODORE L. DeLAND, Treasury Department, Washington, D. C.

Let $2x$ = the rate of interest paid per annum, payable semi-annually; then x = the rate of semi-annual interest, compound. Only \$270,000 was used by the corporation, which was repaid, interest in 10 payments and principal in 5 payments; and they alternate, the first each six months and the second at the end of each year as follows: (1), \$9,000; (2), \$69,000, (3), \$7,200; (4), \$67,200; (5), \$5,400; (6), \$65,400; (7), \$3,600; (8), \$63,600; (9), \$1,800, and (10), \$61,800.

The problem may now be treated, algebraically, as a question in partial payments under the U. S. Court Rule; and we have, after taking out the factor 600, arranging, dropping the dollar sign as follows:

$$450(1+x)^{10} - [15(1+x)^9 + 115(1+x)^8 + 12(1+x)^7 + 112(1+x)^6 \\ + 9(1+x)^5 + 109(1+x)^4 + 6(1+x)^3 + 106(1+x)^2 + 3(1+x)] = 103.$$

Observe that as arranged the first member has 2 terms, and their difference